CS 150, Spring 2006 Quiz 2/B

Name (first last) ....

SID ....

- This exam is **closed book, closed notes** and is designed to take 45 minutes
- Turn off you cell phones
- Write legibly. What can't be read will not be graded
- Good luck!

1		/ 16
2		/ 16
3		/ 18
4		/ 18
5		/ 16
6		/ 16
Total		/100

1. (16 points: 2 points if correct, 1 point if unanswered, 0 points if wrong.)

   Mark by true or false each of the following (no need to prove)

A language cannot be both regular and context free at the same time	□ True	□ False
The family of regular languages is closed under union	□ True	□ False
The grammar $G = (\{S\}, \{\mathtt{a}\}, S, \{S \to \mathtt{a}S \mid S\mathtt{a} \mid \mathtt{a}\})$ is regular	□ True	□ False
The grammar $G = (\{S\}, \{\mathtt{a}, \mathtt{b}\}, S, \{S \to \mathtt{aa}S \mid \mathtt{bb}S \mid \mathtt{b}\})$ is not regular	□ True	□ False
Given a string w and two regular languages L_1 and L_2 there exists an algorithm to determine whether $w \in (L_1 \cap L_2)$	□ True	□ False
The string $(\emptyset(\lambda\emptyset)^*)^*$ is a regular expression	□ True	□ False
The language associated with the regular expression $$(1 + 01)^*(0 + \lambda)$$ is the set of the binary strings containing no occurrences of 00	□ True	□ False
The grammar $G = (\{S, A, B\}, \{\mathtt{a}\}, S, \{S \to AB, A \to \mathtt{a} \mid BA, B \to \lambda \mid AB\})$ is not in Chomsky normal form	□ True	□ False

2. (16 points)

   Write the formal definition of a grammar in Chomsky normal form (CNF).

   Write the formal definition of the language associated with a regular expression.

3. (18 points)

Let $L_1, L_2, \ldots, L_9$ be the following languages over $\Sigma = \{\mathtt{a}, \mathtt{b}\}$

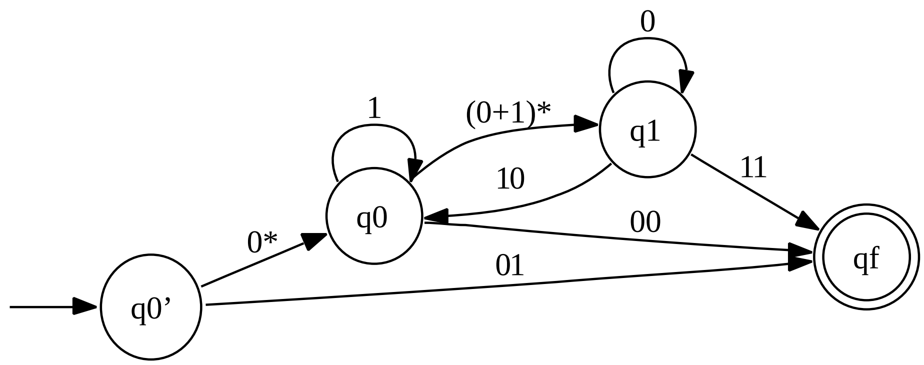
$$\begin{aligned}
L_1 &= \left\{\mathtt{a}^i\mathtt{b}^j : j \geq i \geq 0\right\} \\
L_2 &= \left\{\mathtt{a}^i\mathtt{b}^j : i \geq j \geq 0\right\} \\
L_3 &= \left\{\mathtt{a}^i\mathtt{b}^i\mathtt{a}^j\mathtt{b}^j : i, j \geq 0\right\} \\
L_4 &= \left\{\mathtt{a}^i\mathtt{b}^j\mathtt{a}^j\mathtt{b}^i : i, j \geq 0\right\} \\
L_5 &= \left\{ww^R : w \in \Sigma^*\right\} \\
L_6 &= \left\{\mathtt{a}^i\mathtt{b}^j\mathtt{a}^i : i, j \geq 0\right\} \\
L_7 &= \left\{\mathtt{a}^{i+j}\mathtt{b}^i\mathtt{a}^j : i, j \geq 0\right\} \\
L_8 &= \left\{\mathtt{a}^i\mathtt{b}^j\mathtt{a}^{i+j} : i, j \geq 0\right\} \\
L_9 &= \left\{\mathtt{a}^i\mathtt{b}^{i+j}\mathtt{a}^j : i, j \geq 0\right\}
\end{aligned}$$

In the table below you are given five context-free grammars. For each grammar choose **one** language among $L_1, \ldots, L_9$ generated by this grammar (or write "none" if it does not generate any of these languages).

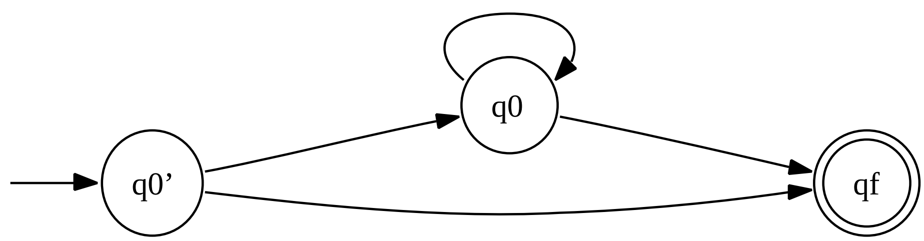
Grammar	*Language*
$S \rightarrow YX$ $X \rightarrow \mathtt{b}X\mathtt{a} \mid \lambda$ $Y \rightarrow \mathtt{a}Y\mathtt{b} \mid \lambda$	
$S \rightarrow \mathtt{a}S\mathtt{b} \mid X$ $X \rightarrow \mathtt{b}X \mid \lambda$	
$S \rightarrow \mathtt{a}S\mathtt{a} \mid X$ $X \rightarrow \mathtt{b}X\mathtt{a} \mid \lambda$	
$S \rightarrow \mathtt{a}S\mathtt{b} \mid X$ $X \rightarrow \mathtt{b}X\mathtt{a} \mid \lambda$	
$S \rightarrow \mathtt{b}S\mathtt{b} \mid \mathtt{a}S\mathtt{a} \mid \lambda$	

4. (18 points)

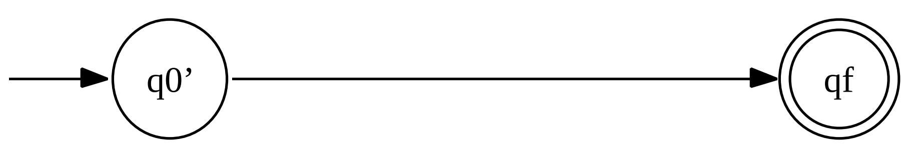
The figure below shows an automaton obtained in the process of converting an NFA to a regular expression. Complete the process by eliminating first q_1 and then q_0.



Write the correct labels on the arcs, after eliminating q_1:



Write the correct labels on the arcs, after eliminating q_0:



5. (16 points)

Prove that the language

$$L = \{\mathtt{a}^n\mathtt{b}^k : n \geq 0, k \geq n\}$$

is not regular. We begin the proof assuming that the opponent has chosen a constant m.

- What string do you choose for $w \in L$, such that $|w| \geq m$?

  $w =$

- Suppose that the adversary decomposes the string $w = xyz$ such that $|xy| \leq m$ and $y \neq \lambda$. What value of i do you choose to create a string $w_i = xy^iz$ such that w_i is not in L?

  $i =$

- Given your choice of i, explain **briefly** why xy^iz does not belong to L.

6. (16 points)

Remove λ-productions, unit-productions, and useless productions, from the following grammar. Show each step of the simplication work.

$$
\begin{aligned}
S &\rightarrow AC \mid C\texttt{a} \\
A &\rightarrow BS \\
B &\rightarrow SA \\
C &\rightarrow \texttt{a}S \mid \lambda
\end{aligned}
$$