CS 150, Spring 2006 Quiz 2/A

Name (first last) ....

SID ....

- This exam is **closed book, closed notes** and is designed to take 45 minutes
- Turn off you cell phones
- Write legibly. What can't be read will not be graded
- Good luck!

1		/ 16
2		/ 16
3		/ 18
4		/ 18
5		/ 16
6		/ 16
Total		/100

1. (16 points: 2 points if correct, 1 point if unanswered, 0 points if wrong.)

Mark by true or false each of the following (no need to prove)

A language can be both regular and context free at the same time	☒ True	☐ False
The family of regular languages is closed under intersection	☒ True	☐ False
The grammar $G = (\{S\}, \{\mathtt{a}\}, S, \{S \to S\mathtt{aaa} \mid \mathtt{a}S \mid \mathtt{a}\})$ is not regular	☒ True	☐ False
The grammar $G = (\{S\}, \{\mathtt{a}, \mathtt{b}\}, S, \{S \to \mathtt{a}S \mid \mathtt{b}S \mid \lambda\})$ is regular	☒ True	☐ False
Given a string w and two regular languages L_1 and L_2 there exists an algorithm to determine whether $w \in (L_1 \cup L_2)$	☒ True	☐ False
The string $(\emptyset(\lambda + \emptyset+)^*)^*$ is a regular expression	☐ True	☒ False
The language associated with the regular expression $(0+1)^*00(0+1)^*$ is the set of the binary strings containing at least one occurrence of 00	☒ True	☐ False
The grammar $G = (\{S, A, B\}, \{\mathtt{a}\}, S, \{S \to AB, A \to \mathtt{a} \mid BA, B \to \lambda \mid AB\})$ is in Chomsky normal form	☐ True	☒ False

2. (16 points)

Write the formal definition of a context free grammar.

Answer: A grammar $G = (V, T, S, P)$ is said to be *context free* if all productions in P have the form $A \to x$ where $A \in V$ and $x \in (V \cup T)^*$.

Write the formal definition of a regular expression.

Answer: Let Σ be a given alphabet. Then

(a) $\emptyset, \lambda$ and $a \in \Sigma$ are all regular expressions.

(b) If r_1 and r_2 are regular expressions, so are $r_1 + r_2$, $r_1 \cdot r_2$, r_1^* and (r_1).

(c) A string is a regular expression if and only if it can be derived using a finite number of applications of the rules in (b) on the expressions listed in (a)

3. (18 points)

Let $L_1, L_2, \ldots, L_9$ be the following languages over $\Sigma = \{\mathtt{a}, \mathtt{b}\}$

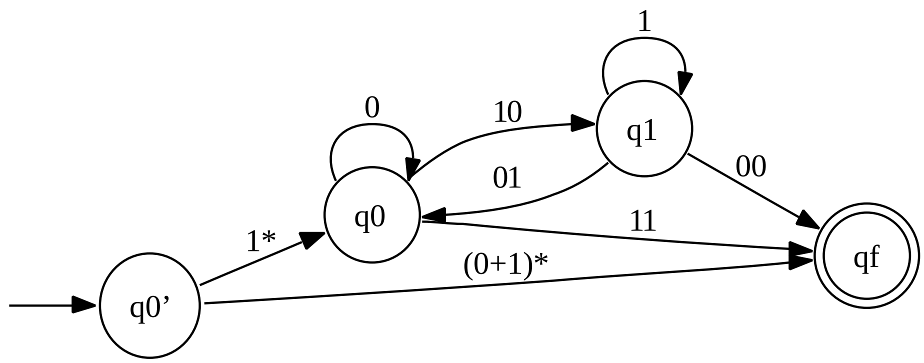
$$
\begin{aligned}
L_1 &= \left\{\mathtt{a}^i\mathtt{b}^j : i \geq j \geq 0\right\} \\
L_2 &= \left\{\mathtt{a}^i\mathtt{b}^j : j \geq i \geq 0\right\} \\
L_3 &= \left\{\mathtt{a}^i\mathtt{b}^j\mathtt{a}^j\mathtt{b}^i : i, j \geq 0\right\} \\
L_4 &= \left\{\mathtt{a}^i\mathtt{b}^i\mathtt{a}^j\mathtt{b}^j : i, j \geq 0\right\} \\
L_5 &= \left\{\mathtt{a}^i\mathtt{b}^j\mathtt{a}^i : i, j \geq 0\right\} \\
L_6 &= \left\{ww^R : w \in \Sigma^*\right\} \\
L_7 &= \left\{\mathtt{a}^i\mathtt{b}^j\mathtt{a}^{i+j} : i, j \geq 0\right\} \\
L_8 &= \left\{\mathtt{a}^i\mathtt{b}^{i+j}\mathtt{a}^j : i, j \geq 0\right\} \\
L_9 &= \left\{\mathtt{a}^{i+j}\mathtt{b}^i\mathtt{a}^j : i, j \geq 0\right\}
\end{aligned}
$$

In the table below you are given five context-free grammars. For each grammar choose **one** language among $L_1, \ldots, L_9$ generated by this grammar (or write "none" if it does not generate any of these languages).

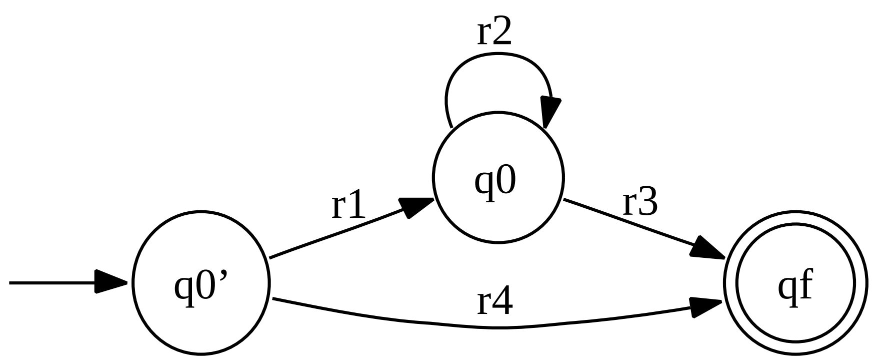
Grammar	*Language*
$S \rightarrow \mathtt{a}S\mathtt{a} \mid \mathtt{b}S\mathtt{b} \mid \lambda$	L_6
$S \rightarrow XY$ $X \rightarrow \mathtt{a}X\mathtt{b} \mid \lambda$ $Y \rightarrow \mathtt{b}Y\mathtt{a} \mid \lambda$	L_8
$S \rightarrow \mathtt{a}S\mathtt{a} \mid X$ $X \rightarrow \mathtt{b}X\mathtt{a} \mid \lambda$	L_7
$S \rightarrow \mathtt{a}S\mathtt{b} \mid X$ $X \rightarrow \mathtt{b}X\mathtt{a} \mid \lambda$	L_3
$S \rightarrow \mathtt{a}S\mathtt{b} \mid X$ $X \rightarrow \mathtt{b}X \mid \lambda$	L_2

4. (18 points)

The figure below shows an automaton obtained in the process of converting an NFA to a regular expression. Complete the process by eliminating first q_1 and then q_0.

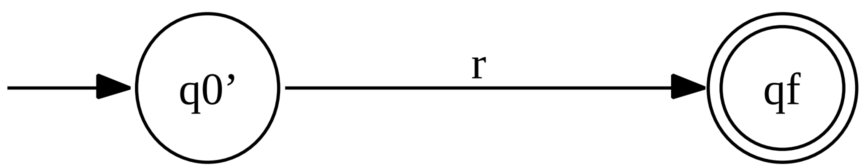


Eliminate q_1:



Where $r_1 = 1^*$, $r_2 = 0 + 101^*01$, $r_3 = 101^*00 + 11$, and $r_4 = (0+1)^*$.

Eliminate q_0:



Where $r = r_1 r_2^* r_3 + r_4$.

5. (16 points)

Prove that the language

$$L = \{\mathtt{a}^n\mathtt{b}\mathtt{a}^n : n \geq 0\}$$

is not regular. We begin the proof assuming that the opponent has chosen a constant m.

- What string do you choose for $w \in L$, such that $|w| \geq m$?

  Answer: $w = \mathtt{a}^m\mathtt{b}\mathtt{a}^m$ (other choices may be also correct).

- Suppose that the adversary decomposes the string $w = xyz$ such that $|xy| \leq m$ and $y \neq \lambda$. What value of i do you choose to create a string $w_i = xy^iz$ such that w_i is not in L?

  Answer: $i = 0$ (other choices may be also correct).

- Given your choice of i, explain **briefly** why xy^iz does not belong to L.

  Answer: The string y is only composed by a's, so that we can write $y = \mathtt{a}^k, k > 0$. By choosing $i = 0$, we get $w_0 = \mathtt{a}^{m-k}\mathtt{b}\mathtt{a}^m$ which does not belong to L because the number of a's before the b is now smaller than the number of a's after the b.

6. (16 points)

Remove λ-productions, unit-productions, and useless productions, from the following grammar. Show each step of the simplication work.

$$\begin{aligned} S &\rightarrow \mathtt{a}C \,|\, CA \\ A &\rightarrow BS \\ B &\rightarrow SA \\ C &\rightarrow S\mathtt{a} \,|\, \lambda \end{aligned}$$

Answer: Remove λ-productions. Since $V_N = \{C\}$, we have

$$\begin{aligned} S &\rightarrow \mathtt{a}C \,|\, \mathtt{a} \,|\, CA \,|\, A \\ A &\rightarrow BS \\ B &\rightarrow SA \\ C &\rightarrow S\mathtt{a} \end{aligned}$$

Remove unit-productions. Since $S \Rightarrow^* A$, we have

$$\begin{aligned} S &\rightarrow \mathtt{a}C \,|\, \mathtt{a} \,|\, CA \,|\, BS \\ A &\rightarrow BS \\ B &\rightarrow SA \\ C &\rightarrow S\mathtt{a} \end{aligned}$$

Remove useless productions. First, reachability. It is easy to verify that you can reach all the variables from S. Next, productivity. We have that $V_1 = \{S, C\}$ (in other words, A and B are non-productive) hence

$$\begin{aligned} S &\rightarrow \mathtt{a}C \,|\, \mathtt{a} \\ C &\rightarrow S\mathtt{a} \end{aligned}$$