1. (16 points: 2 points if correct, 1 point if unanswered, 0 points if wrong.)

Mark by true or false each of the following (no need to prove)

Given a string $u \in \Sigma^+$ we can always find another string $v \in \Sigma^+$, $v \neq u$, such that $uv = vu$	☐ True	☒ False
If L is a language accepted by a NFA A, then there exists a DFA A' that accepts the same language	☒ True	☐ False
Given a DFA $(Q, \Sigma, \delta, q_0, F)$, it may happen that for some $a \in \Sigma$, $\delta^*(q, a) \neq \delta(q, a)$	☐ True	☒ False
The number of outgoing arcs from a state of a DFA is always equal to $\|\Sigma\|$	☒ True	☐ False
The number of outgoing arcs from a state of a NFA is always equal to $\|\Sigma\|$	☐ True	☒ False
Not all finite languages are regular	☐ True	☒ False
If L is regular language, then L^R is also a regular language	☒ True	☐ False
If L is regular language, then L^2 may not be a regular language	☐ True	☒ False

2. (16 points)

   Write the formal definition of a language generated by a grammar.

   Answer: Let $G = (V, T, S, P)$ be a grammar. The language generated by the grammar G is $L(G) = \{w \in T^* : S \Rightarrow^* w\}$.

   Write the formal definition of a language accepted by a DFA.

   Answer: Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA. The language accepted by the DFA M is $L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}$.

3. (16 points)

   Given the following languages over $\Sigma = \{a, b\}$

$$\begin{aligned} L_1 &= \{b^n | n \geq 1\} \\ L_2 &= \{ba^n | n \geq 0\} \\ L_3 &= \{b^n a^n | n \geq 0\} \\ L_4 &= \{(ba)^n | n \geq 1\} \end{aligned}$$

   describe the new languages below using the simplest mathematical notation

   Answer:

   (a) $L_4 \cap \Sigma^* = L_4 = \{(ba)^n | n \geq 1\}$

   (b) $L_3 \cap L_4 = \{ba\}$

   (c) $L_1^2 = \{b^m | m \geq 2\}$

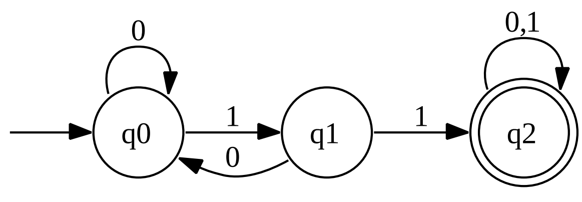
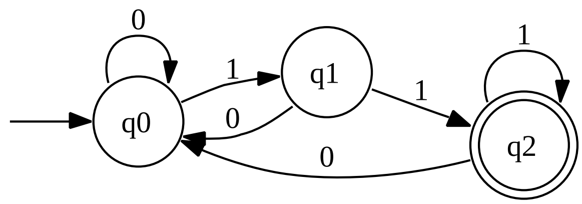
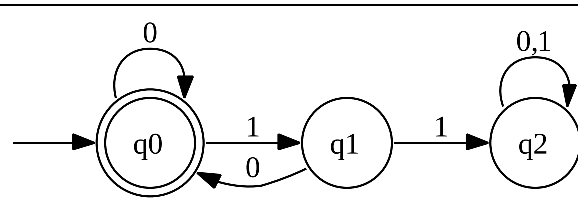
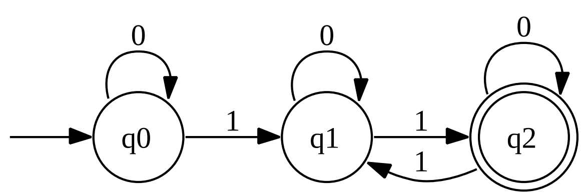
   (d) $L_4 L_4^R = \{(ba)^n (ab)^m | n \geq 1, m \geq 1\}$

4. (18 points)

Let $L_1, L_2, \ldots, L_7$ be the following languages over $\Sigma = \{0, 1\}$

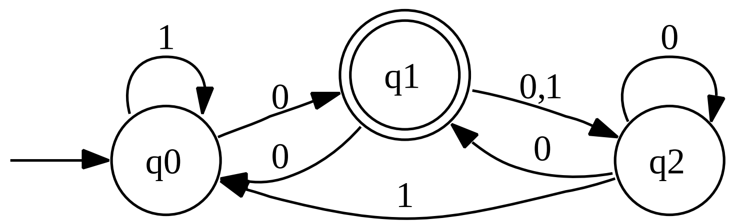
$$
\begin{aligned}
L_1 &= \{w \in \Sigma^* \mid w \text{ ends with } 10\} \\
L_2 &= \{w \in \Sigma^* \mid w \text{ starts with } 11\} \\
L_3 &= \{w \in \Sigma^* \mid w \text{ ends with } 11\} \\
L_4 &= \{w \in \Sigma^* \mid \text{each 1 in } w \text{ is immediately followed by a 0 }\} \\
L_5 &= \{w \in \Sigma^* \mid w \text{ contain the substring } 11\} \\
L_6 &= \{w \in \Sigma^+ \mid w \text{ contains an even number of 1's }\} \\
L_7 &= \{w \in \Sigma^* \mid w \text{ contains an even number of 1's}\}
\end{aligned}
$$

For each DFA shown below, tell which of the languages above it accepts (write NONE if none of the above matches the language accepted by the DFA)

automaton	language
	L_5
	L_3
	L_4
	NONE

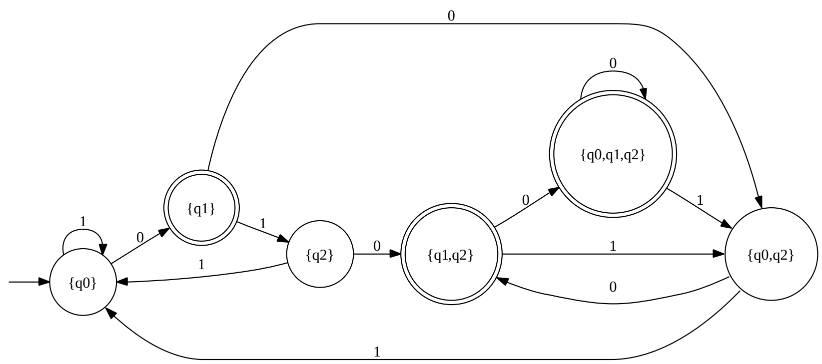
5. (18 points)

Let A be the following NFA



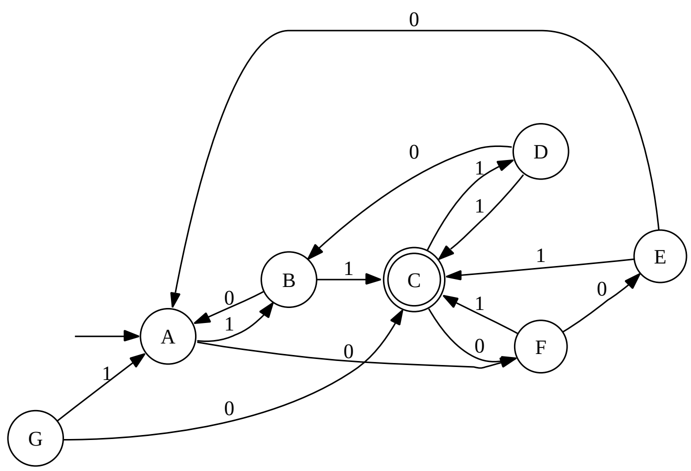
Draw the DFA equivalent to A.

Answer:



6. (16 points)

Let A be the following DFA



and let

B	x				
C	x	x			
D	x	x	x		
E	?		x	x	
F	x	x	x		?
	A	B	C	D	E

be an intermediate table of distinguished states produced by the algorithm MINIMIZE_DFA(A) described in class.

- Complete the table by checking whether the pairs of states marked with `?` are distinguishable (write `x`) or not (draw a circle around the `?`)

  Answer:

B	x				
C	x	x			
D	x	x	x		
E	x		x	x	
F	x	x	x		x
	A	B	C	D	E

- Draw the graph for the minimal DFA $\hat{A}$.

  Answer: