

Name (first last) .....

SSN .....

- This exam is **closed book, closed notes** and is designed to take 45 minutes
- Please turn off you cell phones
- Write legibly. What can't be read will not be graded
- Good luck!

1	<input type="text"/>	/16
2	<input type="text"/>	/16
3	<input type="text"/>	/16
4	<input type="text"/>	/18
5	<input type="text"/>	/18
6	<input type="text"/>	/16
Total	<input type="text"/>	/100

1. (16 points: 2 points if correct, 1 point if unanswered, 0 points if wrong.)

Mark by true or false each of the following (no need to prove)

Given a string  $u \in \Sigma^+$  we can always find another string  $v \in \Sigma^+$ ,  $v \neq u$ , such that  $uv = vu$   True  False

If  $L$  is a language accepted by a NFA  $A$ , then there exists a DFA  $A'$  that accepts the same language  True  False

Given a DFA  $(Q, \Sigma, \delta, q_0, F)$ , it may happen that for some  $a \in \Sigma$ ,  $\delta^*(q, a) \neq \delta(q, a)$   True  False

The number of outgoing arcs from a state of a DFA is always equal to  $|\Sigma|$   True  False

The number of outgoing arcs from a state of a NFA is always equal to  $|\Sigma|$   True  False

Not all finite languages are regular  True  False

If  $L$  is regular language, then  $L^R$  is also a regular language  True  False

If  $L$  is regular language, then  $L^2$  may not be a regular language  True  False

2. (16 points)

Write the formal definition of a language generated by a grammar.

Write the formal definition of a language accepted by a DFA.

3. (16 points)

Given the following languages over  $\Sigma = \{a, b\}$

$$L_1 = \{b^n | n \geq 1\}$$

$$L_2 = \{ba^n | n \geq 0\}$$

$$L_3 = \{b^n a^n | n \geq 0\}$$

$$L_4 = \{(ba)^n | n \geq 1\}$$

describe the new languages below using the simplest mathematical notation

(a)  $L_4 \cap \Sigma^* = \{ \}$

(b)  $L_3 \cap L_4 = \{ \}$

(c)  $L_1^2 = \{ \}$

(d)  $L_4 L_4^R = \{ \}$

4. (18 points)

Let  $L_1, L_2, \dots, L_7$  be the following languages over  $\Sigma = \{0, 1\}$

$$L_1 = \{w \in \Sigma^* \mid w \text{ ends with } 10\}$$

$$L_2 = \{w \in \Sigma^* \mid w \text{ starts with } 11\}$$

$$L_3 = \{w \in \Sigma^* \mid w \text{ ends with } 11\}$$

$$L_4 = \{w \in \Sigma^* \mid \text{each } 1 \text{ in } w \text{ is immediately followed by a } 0\}$$

$$L_5 = \{w \in \Sigma^* \mid w \text{ contain the substring } 11\}$$

$$L_6 = \{w \in \Sigma^+ \mid w \text{ contains an even number of } 1\text{'s}\}$$

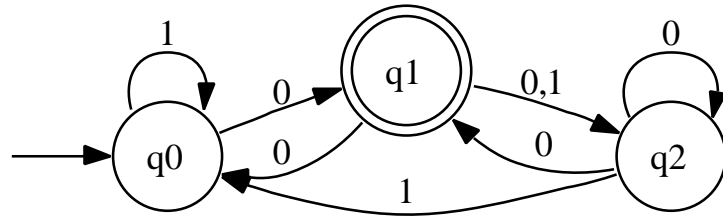
$$L_7 = \{w \in \Sigma^* \mid w \text{ contains an even number of } 1\text{'s}\}$$

For each DFA shown below, tell which of the languages above it accepts (write NONE if none of the above matches the language accepted by the DFA)

automaton	language
<pre> graph LR     start(( )) --&gt; q0((q0))     q0 -- 0 --&gt; q0     q0 -- 1 --&gt; q1((q1))     q1 -- 0 --&gt; q0     q1 -- 1 --&gt; q2(((q2)))     q2 -- "0,1" --&gt; q2         </pre>	
<pre> graph LR     start(( )) --&gt; q0((q0))     q0 -- 0 --&gt; q0     q0 -- 1 --&gt; q1((q1))     q1 -- 0 --&gt; q0     q1 -- 1 --&gt; q2(((q2)))     q2 -- 0 --&gt; q0     q2 -- 1 --&gt; q2         </pre>	
<pre> graph LR     start(( )) --&gt; q0(((q0)))     q0 -- 0 --&gt; q0     q0 -- 1 --&gt; q1((q1))     q1 -- 0 --&gt; q0     q1 -- 1 --&gt; q2((q2))     q2 -- "0,1" --&gt; q2         </pre>	
<pre> graph LR     start(( )) --&gt; q0((q0))     q0 -- 0 --&gt; q0     q0 -- 1 --&gt; q1((q1))     q1 -- 0 --&gt; q1     q1 -- 1 --&gt; q2(((q2)))     q2 -- 1 --&gt; q1     q2 -- 0 --&gt; q2         </pre>	

5. (18 points)

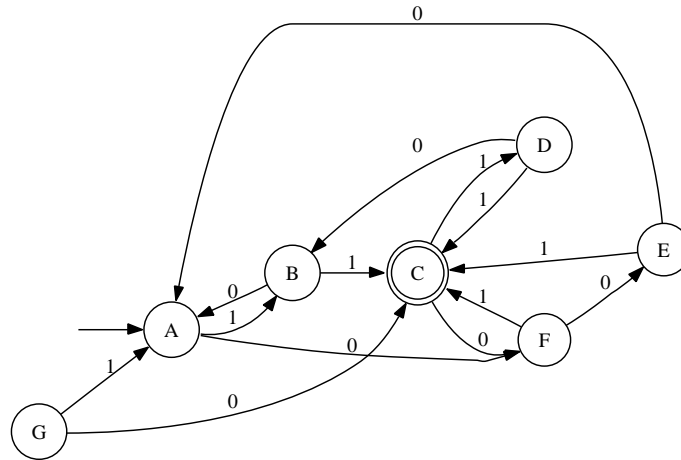
Let  $A$  be the following NFA



Draw the DFA equivalent to  $A$ .

6. (16 points)

Let  $A$  be the following DFA



and let

B	x				
C	x	x			
D	x	x	x		
E	?		x	x	
F	x	x	x		?
	A	B	C	D	E

be an intermediate table of distinguished states produced by the algorithm  $\text{MINIMIZE\_DFA}(A)$  described in class.

- Complete the table by checking whether the pairs of states marked with ? are distinguishable (write  $x$ ) or not (draw a circle around the ?)
- Draw the graph for the minimal DFA  $\hat{A}$ .