1. (16 points: 2 points if correct, 1 point if unanswered, 0 points if wrong.)

Mark by true or false each of the following (no need to prove)

Given a string $u \in \Sigma^+$ we can always find another string $v \in \Sigma^*$, $v \neq u$, such that $uv = vu$	☒ True	☐ False
There exists at least one language L accepted by a NFA for which we cannot produce a DFA that accepts it	☐ True	☒ False
Given a DFA $(Q, \Sigma, \delta, q_0, F)$, we have that for all $a \in \Sigma$, $\delta^*(q, a) = \delta(q, a)$	☒ True	☐ False
The number of incoming arcs to a state of a DFA is always equal to $\|\Sigma\|$	☐ True	☒ False
The number of outgoing arcs from a state of a NFA is always less than or equal to $\|\Sigma\| + 1$	☐ True	☒ False
All finite languages are regular	☒ True	☐ False
If L is a regular language, then L^R may not be a regular language	☐ True	☒ False
If L is a regular language, then L^2 is also a regular language	☒ True	☐ False

2. (16 points)

   Write the formal definition of a language accepted by a NFA.

   Answer: Let $M = (Q, \Sigma, \delta, q_0, F)$ be a NFA. The language accepted by the NFA M is $L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \cap F \neq \emptyset\}$.

   Write the formal definition of a regular language.

   Answer: A language L is regular if there exists a DFA M such that accepts $L(M) = L$.

3. (16 points)

   Given the following languages over $\Sigma = \{a, b\}$

$$
\begin{aligned}
L_1 &= \{b^n | n \geq 1\} \\
L_2 &= \{ba^n | n \geq 0\} \\
L_3 &= \{b^n a^n | n \geq 0\} \\
L_4 &= \{(ba)^n | n \geq 1\}
\end{aligned}
$$

   describe the new languages below using the simplest mathematical notation

   Answer:

   (a) $L_1 \cap \Sigma^* = L_1 = \{b^n | n \geq 1\}$

   (b) $L_2 \cap L_3 = \{ba\}$

   (c) $L_4^2 = \{(ba)^m | m \geq 2\}$

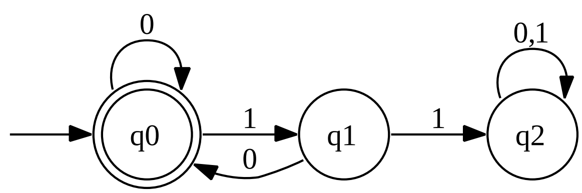
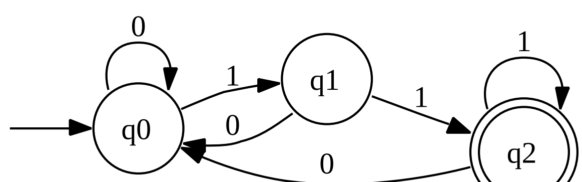
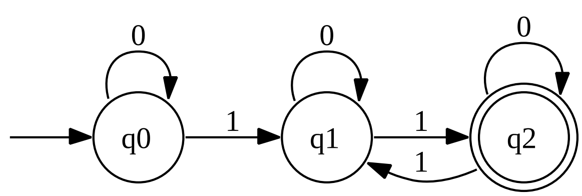
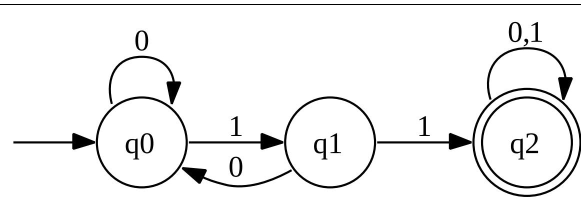
   (d) $L_3 L_3^R = \{b^n a^{n+m} b^m | n \geq 0, m \geq 0\}$

4. (18 points)

Let $L_1, L_2, \ldots, L_7$ be the following languages over $\Sigma = \{0, 1\}$

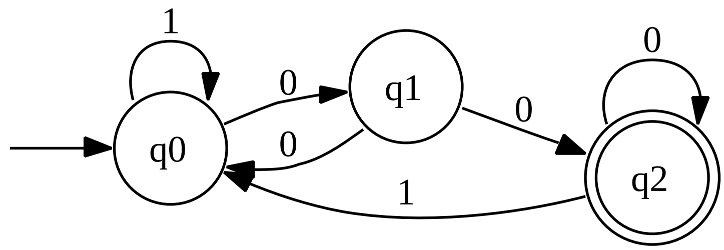
$$
\begin{aligned}
L_1 &= \{w \in \Sigma^* \mid w \text{ ends with } 11\} \\
L_2 &= \{w \in \Sigma^* \mid w \text{ starts with } 11\} \\
L_3 &= \{w \in \Sigma^* \mid w \text{ ends with } 10\} \\
L_4 &= \{w \in \Sigma^* \mid w \text{ contain the substring } 11\} \\
L_5 &= \{w \in \Sigma^* \mid \text{each 1 in } w \text{ is immediately followed by a 0 }\} \\
L_6 &= \{w \in \Sigma^* \mid w \text{ contains an even number of 1's}\} \\
L_7 &= \{w \in \Sigma^+ \mid w \text{ contains an even number of 1's }\}
\end{aligned}
$$

For each DFA shown below, tell which of the languages above it accepts (write NONE if none of the above matches the language accepted by the DFA)

automaton	language
	L_5
	L_1
	NONE
	L_4

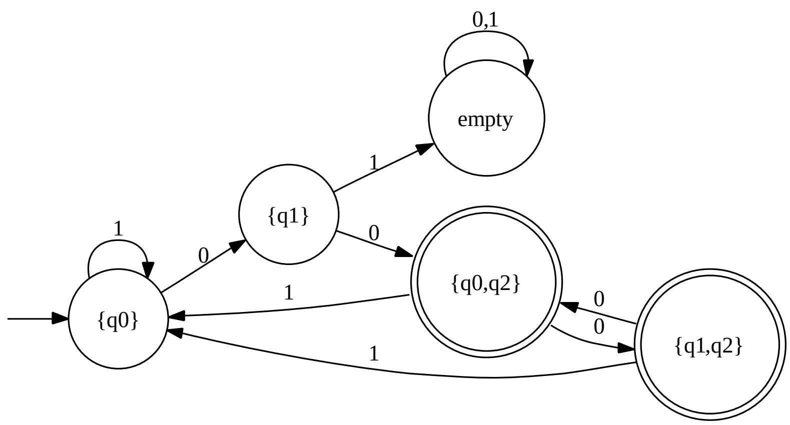
5. (18 points)

Let A be the following NFA



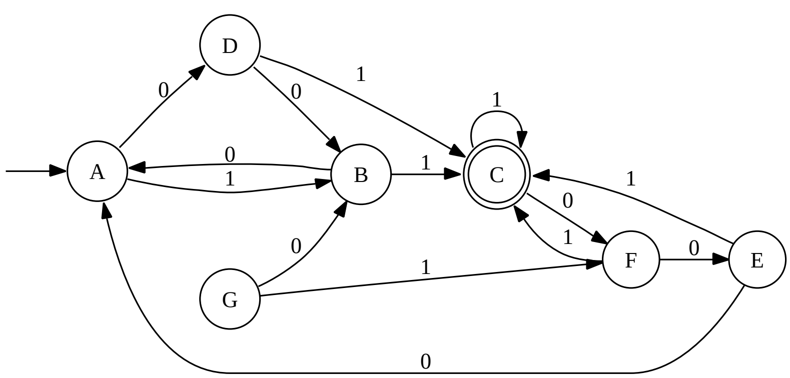
Draw the DFA equivalent to A.

Answer:



6. (16 points)

Let A be the following DFA



and let

B	?				
C	x	x			
D	x	?	x		
E	x		x	x	
F	x	x	x		x
	A	B	C	D	E

be an intermediate table of distinguished states produced by the algorithm MINIMIZE_DFA(A) described in class.

- Complete the table by checking whether the pairs of states marked with `?` are distinguishable (write `x`) or not (draw a circle around the `?`)

  Answer:

B	x				
C	x	x			
D	x	x	x		
E	x		x	x	
F	x	x	x		x
	A	B	C	D	E

- Draw the graph for the minimal DFA $\hat{A}$.

  Answer: